Random Graphs
Exercise Sheet 1

Question 1. Give an example of a set of events $\{A_i : i \in I\}$ which are pairwise independent, but not mutually independent.

Give an example of a collection of random variables $\{X_i : i \in I\}$ which are pairwise independent, but not mutually independent.

Give an example of two random variables X and Y such that $\mathbb{E}(XY) \neq \mathbb{E}(X)\mathbb{E}(Y)$.

Question 2. Calculate the following probabilities:

- $\mathbb{P}(G(4, 1/2)$ has 2 edges);
- $\mathbb{P}(G(4, 1/2)$ has 6 edges);
- $\mathbb{P}(G(4, 1/2)$ is connected).

Question 3. Let $(G(1), \ldots, G(\binom{n}{2}))$ be the sequence of random variables given by the random graph process in the lecture. Show that $G(m) \sim G_{n,m}$ for each m.

Question 4. Show that with high probability $\mathbb{P}(G(n, p)$ has diameter $\leq 2)$, for constant p.

Question 5. Let G be a graph. Show that G contains an independent set of size at least

$$\sum_{x \in V} \frac{1}{d(v) + 1}.$$

Question 6. Show directly, that is without using any relation between $G_{n,m}$ and $G_{n,p}$, that every monotone graph property has a threshold in $G_{n,m}$.

(Hint: It may be useful to relate the random variable $G_{n,km}$ to the random variable given by the union of k independent copies of $G_{n,m}$).